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Induced noise control

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Abstract

A technique by which to achieve noise control in a dynamic system, referred to herein as a master dynamic system, is to couple it to another, an adjunct dynamic system. The resulting induced noise control is defined in terms of the ratio of the energy stored in the master dynamic system when coupled to the adjunct dynamic system to the stored energy in the master dynamic system in the absence of this coupling. An analytical description for the induced noise control is developed. The analysis is then used to define the dependence of the induced noise control on the parameters that define the two dynamic systems and the coupling between them. An appropriate induced noise control may then be designed and implemented. Also, the relationship of this energy analysis to the statistical energy analysis is briefly discussed.
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1. Introduction

The stored energy in a dynamic system—the *master* dynamic system—may be altered by coupling it to another. The dynamic system that is coupled to the master dynamic system is dubbed the *adjunct* dynamic system [1–12]. Since the energy stored in a dynamic system is an eligible measure of its (quadratic average) response, an appropriately designed adjunct dynamic system may induce a reduction in the response of the master dynamic system. This reduction in the response is the *induced* noise control [1–12]. In that vein an *induced* noise control parameter is defined in terms of the ratio of the stored energy in a master dynamic system when it is coupled to an adjunct dynamic system, to that stored energy when the coupling is absent. The induced noise control parameter so defined comprises of two factors. The first is the ratio of the *indigenous* loss factor of the master dynamic system to its *virtual* loss factor. The virtual loss factor is the sum of the indigenous loss factor and the *induced* loss factor in the master dynamic system. The induced loss factor measures the change in the loss factor in the master dynamic system due to its coupling

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to the adjunct dynamic system. Since the loss factors pertaining to passive dynamic systems are positive definite, the first factor, is, by definition, less than unity. Thus, this first factor contributes beneficially to the induced noise control parameter. The second factor is the ratio of the external input power into the master oscillator when coupled, to the external input power in the absence of this coupling. This second factor, which is also positive definite, may either exceed unity, be unity, or be less than unity [14–16]. Both factors, in the induced noise control parameter, are critically dependent on the *global coupling strength* and are, therefore, interdependent. The global coupling strength is itself a ratio. This ratio is that of the energy stored in the adjunct dynamic system and in the coupling, to the energy stored in the master dynamic system. The investigation of this ratio and its relationship to the induced loss factor is explained and is graphically illustrated.

The ratio of the *modal density* in the adjunct dynamic system to that in the master dynamic system defines, together with the global coupling strength, a *modal coupling strength*. The significance of the modal coupling strength is that in the statistical energy analysis (SEA) the value of the modal coupling strength is, by definition, less than unity [1–4,17–19]. In the energy analysis (EA), herein developed, the modal coupling strength may exceed unity. This excess occurs in the energy analysis (EA) when the *coupling is strong* and the *damping* assigned to the adjunct dynamic system *is low*. The damping in the adjunct dynamic system is conveniently defined in terms of an associated *modal overlap parameter* [3,4]. To reconcile the statistical energy analysis (SEA) with the energy analysis (EA) in focus, the associated modal overlap parameter in the adjunct dynamic system must necessarily exceed a threshold. This restriction has not been previously recognized by most (SEA) practitioners.

2. The concept and the analytical definition of the induced noise control parameter

Damping treatment is often employed in order to diminish the response of an externally force-driven reverberant dynamic system. The statement is linked to the definition of damping in terms of the loss factor $\eta_0(\omega)$ namely

$$\eta_0(\omega) = [\Pi_e^0(\omega)/\omega E_0^0(\omega)], \qquad \Pi_e^0(\omega) = \eta_0(\omega)[\omega E_0^0(\omega)], \tag{1}$$

where $\Pi_e^0(\omega)$ is the power input from the external force-drive, $E_0^0(\omega)$ is the stored energy in the dynamic system and (ω) is the frequency variable [1–4]. The stored energy in a dynamic system is an eligible measure of its (quadratic average) response; see Fig. 1, [20]. From Eq. (1) it follows that increasing the damping, which entails an increase in the loss factor, will result in a decrease in the response.

Utilizing Eq. (1), a stored energy reduction scheme may be proposed: the dynamic system in focus—the *master* dynamic system—is modified; e.g., by appropriately coupling it to another; see Fig. 2. The *adjunct* dynamic system, to which the master dynamic system is to be coupled, is suitably designed to change the loss factor as perceived by the external force-drive, from $\eta_0(\omega)$ to $\eta_v(\omega)$. In the modified dynamic system, as depicted in Fig. 2(b) and in the vein of Eq. (1), one may define

$$\eta_v(\omega) = [\Pi_e(\omega)/\omega E_0(\omega)], \qquad \Pi_e(\omega) = \eta_v(\omega)[\omega E_0(\omega)], \tag{2}$$

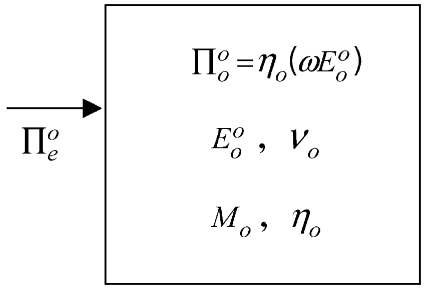

Fig. 1. An externally force-driven isolated master dynamic system. Subscript (o) designates quantities and parameters that pertain to the master dynamic system. Superscript (o) designates quantities and parameters that pertain to the uncoupled master dynamic system. Π_e^0 = input power generated by an external force-drive. E_0^0 = stored energy = $N_0\varepsilon_0^0$; ε_0^0 = modal stored energy. N_0 = number of modes = $\Delta(\omega)\nu_0$; ν_0 = modal density. M_0 = global mass = $N_0 m_0$; m_0 = modal mass. η_0 = indigenous loss factor; ω = frequency variable. $\Delta(\omega)$ = frequency bandwidth centered about (ω).

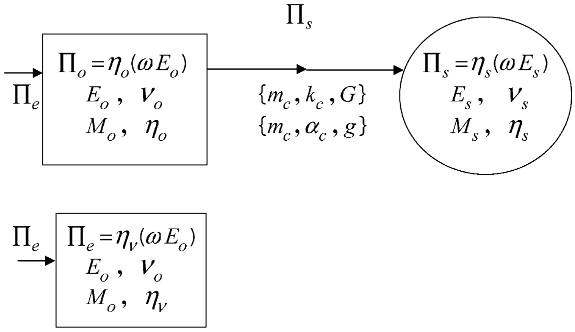

Fig. 2. An externally force-driven master dynamic system passively coupled to an adjunct dynamic system. Subscripts (0) and (s) designate quantities and parameters that pertain to the master dynamic system and to the adjunct dynamic system, respectively. (a) Both dynamic systems are viewed. Π_s = the net power that is imparted to the adjunct dynamic system from the master dynamic system.

The vector $\{m_c, k_c, G\}$ and its normalized form $\{\bar{m}_c, \alpha_c, g\}$ describe the coupling coefficients {mass, stiffness and gyroscopic} between the two dynamic systems (cf. Fig. 1). Π_e = input power generated by an external force-drive. E_0 = stored energy in the master dynamic system. E_s = stored energy in the adjunct dynamic system. N_s = number of modes in adjunct dynamic system = $\Delta(\omega)\nu_s$; ν_s = modal density in adjunct dynamic system. η_s = indigenous loss factor in the adjunct dynamic system. M_s = global mass of the adjunct dynamic system = $N_s m_s$; m_s = modal mass in the adjunct dynamic system. (b) Only the master dynamic system is viewed. η_v = virtual loss factor.

where the corresponding changes in the stored energy and in the external input power are from $E_0^0(\omega)$ to $E_0(\omega)$ and from $\Pi_e^0(\omega)$ to $\Pi_e(\omega)$, respectively [1–19]. The external force-drive in this consideration is assumed to remain unchanged under the modification to the dynamic system as just proposed. The *induced* noise control parameter, designated $\xi(\omega)$, may then be expressed, from Eqs. (1) and (2), in the form

$$\xi(\omega) = [E_0(\omega)/E_0^0(\omega)] = \xi_0(\omega)\pi(\omega), \qquad \xi_0(\omega) = [\eta_0(\omega)/\eta_v(\omega)], \tag{3}$$

where $\xi_0(\omega)$ is the ratio of the loss factors of the master dynamic system before to that after coupling and the external input power ratio $\pi(\omega)$ is defined as

$$\pi(\omega) = [\Pi_e(\omega)/\Pi_e^0(\omega)]. \tag{4}$$

Employing Eqs. (3) and (4) one may speculate that if the external input power is uninfluenced by the modification, i.e., if

$$\pi(\omega) \equiv 1, \tag{5a.1}$$

and if in addition, the modification successfully achieves a loss factor $\eta_v(\omega)$ such that it significantly exceeds the indigenous loss factor $\eta_0(\omega)$, then,

$$\xi(\omega) \Rightarrow \xi_0(\omega) = [\eta_0(\omega)/\eta_v(\omega)] \ll 1 \tag{5b}$$

and consequently the induced noise control is highly beneficial; namely, the noise control parameter $\xi(\omega)$ is small compared with unity, $\xi(\omega) \ll 1$. The condition stated in Eq. (5a.1) is often assumed by noise control engineers to be true, and Eq. (5b) is thus assumed valid. However, here this assumption is challenged; Eq. (5a.1) may not hold. After all, a change in the loss factor, perceived by an external force-drive in a dynamic system, may, under certain conditions, be expected to change the external input power. Then, increasing the degree of damping, in the manner here prescribed and stated in Eq. (5b), may result in an input power ratio $\pi(\omega)$ which may either stay as stated in Eq. (5a.1), exceed unity or fall below unity:

$$\pi(\omega) > 1, \tag{5a.2}$$

or

$$\pi(\omega) < 1. \tag{5a.3}$$

In the absence of heavy handedness in the construction of an adjunct dynamic system, $\pi(\omega)$ usually exceeds unity, i.e., Eq. (5a.2) is usually valid. Then the highly beneficial noise control that Eqs. (5a.1) and (5b) are promising, is mollified as

$$\xi(\omega) = \xi_0(\omega)\pi(\omega) > [\eta_0(\omega)/\eta_v(\omega)] = \xi_0(\omega), \tag{5c}$$

where use is made of Eq. (3) [14]. The principle that underlines this mollifying influence—the mitigator of many a noise control panacea—is attributed to Chatelier. Here the manifestation of Le Chatelier's Principle reads: An externally driven master dynamic system, in equilibrium, is coupled to an energy storing (reverberant) adjunct dynamic system with the intention of increasing the loss factor of the master dynamic system. With the external drive intact, the external input power into the coupled dynamic systems, in equilibrium, increases, thereby (partially or even amply) counteracting the benefit accrued from the resulting increase in the loss factor [15–22].

Focusing attention on Fig. 2, the relationships that are governed by the *conservation of energy* are employed to define a number of loss factors and the *global* and *modal* coupling strengths. With the appropriate interpretation of these definitions one may derive an estimation of the external input power ratio $\pi(\omega)$ and, hence, provide a more definitive estimate for this quantity than Eq. (5c) does [14–16]. Then, the induced noise control parameter $\xi(\omega)$, stated in Eq. (3), may be properly estimated. In this paper the estimates are cast predominately in terms of mean values, as prescribed by Skudrzyk, and computational illustrations of these estimates are cited [20].

3. Relationships stemming from the conservation of energy

Focusing attention on Fig. 2, one may supplement Eq. (2) with the following definitions derived off the conservation of energy (power). These definitions may be expressed in the forms

$$\Pi_0(\omega) = \eta_0(\omega)[\omega E_0(w)], \qquad \Pi_s(\omega) = \eta_I(\omega)[\omega E_0(\omega)] = \eta_s(\omega)[\omega E_s(\omega)],$$
$$\Pi_e(\omega) = \Pi_0(\omega) + \Pi_s(\omega), \qquad \Pi_e(\omega) = \eta_v(\omega)[\omega E_0(\omega)], \tag{6a}$$

and hence

$$\eta_v(\omega) = \eta_0(\omega) + \eta_I(\omega), \qquad \xi_0(\omega) = [1 + \{\eta_I(\omega)/\eta_0(\omega)\}]^{-1},$$
$$\eta_I(\omega) = \eta_s(\omega)\Im_0^s(\omega), \qquad \Im_0^s(\omega) = [E_s(\omega)/E_0(\omega)], \tag{6b}$$

where $\Pi_0(\omega)$ is the portion of the external input power dissipated in the master dynamic system, and $\Pi_s(\omega)$, $E_s(\omega)$ and $\eta_s(\omega)$ are the portion of the external input power dissipated, the energy stored and the loss factor in the adjunct dynamic system, respectively, and, finally, $\Im_0^s(\omega)$ is the ratio of the energy stored in the adjunct dynamic system to that in the master dynamic system [1–4]. (The coupling elements are considered as part of the adjunct dynamic system, i.e., the stored energy $E_s(\omega)$ includes the stored energy that resides in the coupling elements.) The stored energy ratio $\Im_0^s(\omega)$ is dubbed the *global* coupling strength between the adjunct and the force-driven master dynamic systems. The loss factor $\eta_I(\omega)$ is dubbed the *induced* loss factor; the loss factor that is induced in the master dynamic system by virtue of its coupling to the adjunct dynamic system [5–19,23–25]. In the absence of coupling $\eta_I(\omega)$ is identically equal to zero. In addition to the *virtual* loss factor $\eta_v(\omega)$, defined and stated in Eqs. (2) and (6), an *effective* loss factor $\eta_e(\omega)$ of the *combined*, master + adjunct, dynamic systems may be introduced. This effective loss factor is defined in the form

$$\Pi_e(\omega) = \eta_e(\omega)[\omega E(\omega)], \qquad \eta_e(\omega) = [\Pi_e(\omega)/\omega E(\omega)], \qquad E(\omega) = E_0(\omega) + E_s(\omega),$$
$$[\Pi_e(\omega)/\omega E_0(\omega)] = \eta_v(\omega) = \eta_e(\omega)[1 + \Im_0^s(\omega)], \qquad \Im_0^s(\omega) = [E_s(\omega)/E_0(\omega)] \tag{7a}$$

and it follows that

$$[\eta_v(\omega)/\eta_e(\omega)] = [1 + \Im_0^s(\omega)] \geqslant 1. \tag{7b}$$

In Figs. 1 and 2 the modal densities are given as parameters of significance. The significance of these parameters emerges when it becomes convenient and instructive to define *modal* quantities from *global* counterparts. Of particular interest herein are the definitions of the averaged modal coupling strength $\varsigma_0^s(\omega)$ and the modal mass ratio (m_s/m_0). These modal quantities are related to the corresponding global quantities in the form

$$\Im_0^s(\omega) = [v_s(\omega)/v_0(\omega)] \quad \varsigma_0^s(\omega), \tag{8a}$$

$$(M_s/M_0) = [v_s(\omega)/v_0(\omega)] \quad (m_s/m_0), \tag{8b}$$

where $v_0(\omega)$ and $v_s(\omega)$ are the *modal densities* in the master dynamic system and the adjunct dynamic system, respectively [3,4]. The modal densities also feature prominently when one wishes to estimate the external input power $\Pi_e(\omega)$ into a dynamic system. This estimate yields

$$\Pi_e(\omega) = S_f(\omega)G(\omega), \qquad G(\omega) = (\pi/2)[v(\omega)/M], \qquad S_f(\omega) = \Delta\omega s_f(\omega), \tag{9}$$

where $s_f(\omega)$ is the power spectral density of the external force-drive [3,4]. In Eq. (9), $v(\omega)$ is the modal density and (M) is the mass of the dynamic system as perceived by the external force-drive [3,4].[1] In Eq. (9), $(\Delta\omega)$ is a suitable frequency bandwidth centered about the frequency (ω) [3,4]. From Fig. 1 and Eq. (9) the external input power, $\Pi_e^0(\omega)$, into the master dynamic system in the absence of coupling may then be stated in the form

$$\Pi_e^0(\omega) \cong S_f(\omega)(\pi/2)[v_0(\omega)/M_0], \tag{10a}$$

where $v_0(\omega)$ and (M_0) are the modal density and mass of the master dynamic system, respectively [3,4]. The modal density $v_0(\omega)$ and the mass (M_0), perceived by the external force-drive in the absence of coupling, may be modified by the presence of the coupling. The modified modal density and the mass are designated by $v_0^s(\omega)$ and $M_0^s(\omega)$, respectively. In this event Eq. (9) assumes the form

$$\Pi_e(\omega) = S_f(\omega)(\pi/2)[v_0^s(\omega)/M_0^s(\omega)]. \tag{10b}$$

It is speculated that the modified modal density $v_0^s(\omega)$ and mass $M_0^s(\omega)$ are related to the global and the modal coupling strength, $\Im_0^s(\omega)$ and $\varsigma_0^s(\omega)$ in the forms

$$v_0^s(\omega) = [v_0(\omega) + \varsigma_0^s(\omega)v_s(\omega)] = v_0(\omega)[1 + \Im_0^s(\omega)], \tag{11a}$$

$$M_0^s(\omega) = M_0[1 + F\{(M_s/M_0)\varsigma_0^s(\omega)\}] = M_0[1 + F\{(m_s/m_0)\Im_0^s(\omega)\}] \tag{11b}$$

where (m_0) and (m_s) are the *modal* masses in the master dynamic system and in the adjunct dynamic system, respectively. Clearly, Eq. (11a) is less problematic than Eq. (11b) [3,4,14]. Indeed, the functional form of (F) is not readily established unless $\{(M_s/M_0)\varsigma_0^s(\omega)\}$ is small, compared with unity; in this event, $F\{(M_s/M_0)\varsigma_0^s\}$ is comparably small, i.e.,

$$F\{(M_s/M_0)\varsigma_0^s\} \cong \{(M_s/M_0)\varsigma_0^s\} \ll 1. \tag{11c}$$

Eqs. (11a) and (11c) are then compatible. If the power spectral density $s_f(\omega)$ of the external force-drive remains intact when coupling is instituted, one derives from Eqs. (9)–(11), for the external input power $\Pi_e(\omega)$ into the coupled master dynamic system, the form

$$\Pi_e(\omega) \cong \Pi_e^0(\omega)[1 + \Im_0^s(\omega)][1 + F\{(m_s/m_0)\Im_0^s(\omega)\}]^{-1}, \quad S_f(\omega) = \Delta\omega s_f(\omega). \tag{10c}$$

From Eqs. (3)–(11) one obtains finally,

$$\xi(\omega) = \xi_0(\omega)\pi(\omega), \tag{12a.1}$$

$$\xi_0(\omega) = [\eta_0(\omega)/\eta_v(\omega)] = [1 + \{\eta_s(\omega)/\eta_0(\omega)\}\Im_0^s(\omega)]^{-1} < 1, \tag{12b}$$

[1] Without belaboring the subject matter beyond the scope here considered, it is to be understood that one may visualize special descriptions, for the external force-drive, the master structure and the coupling to the adjunct structure, to which Eq. (9) may not apply. Indeed, in a special description of this kind, the ratio $\pi(\omega)$ of the external input power may well nigh be equal to unity and Eqs. (5a.1) and (5b) reign supreme. Then much of the subsequent developments and conclusions utilizing $\pi(\omega) \neq 1$ become moot; e.g., the induced noise control reversal and the idiosyncrasy between the energy analysis (EA) and the statistical energy analysis (SEA) vanish. In this paper, however, the validity of Eq. (9) is not questioned.

$$\pi(\omega) = [1 + \Im_0^s(\omega)][1 + F\{(m_s/m_0)\Im_0^s(\omega)\}]^{-1}. \tag{12c.1}$$

In the more usual noise control designs, the modal mass ratio (m_s/m_0) is small enough to render $[(m_s/m_0)\Im_0^s(\omega)]$ small compared with unity even when the stored energy ratio $\Im_0^s(\omega)$ may far exceed unity. The exceptions are found in the design of a light panel (the master dynamic system) that is intended to store heavy electronic components (constituting collectively the adjunct dynamic system) in sections of space crafts [26]. Excluding these exceptions in subsequent considerations, it is to be assumed that $[(m_s/m_0)\Im_0^s(\omega)]$ is small compared with unity and, therefore, in subsequent considerations the factor $[1 + F\{(m_s/m_0)\Im_0^s(\omega)\}]$ is approximated as equal to unity. Under this mass condition, Eqs. (12a.1) and (12c.1) may be explicitly approximated

$$\xi(\omega) = [1 + \{\eta_s(\omega)/\eta_0(\omega)\}\Im_0^s(\omega)]^{-1}[1 + \Im_0^s(\omega)], \tag{12a.2}$$

$$\pi(\omega) = [\Pi_e(\omega)/\Pi_e^0(\omega)] = [\eta_v(\omega)/\eta_e(\omega)] = [1 + \Im_0^s(\omega)] > 1, \tag{12c.2}$$

respectively, where the effective loss factor $\eta_e(\omega)$ is defined in Eq. (7). Under this imposition Eq. (12) already poses a critical question: may an adjunct dynamic system, that is destined to be passively coupled to an externally force-driven master dynamic system, be appropriately designed to achieve a credible noise control in that master dynamic system? Three cases are detailed to exemplify three specific but diverse answers to this question.

Case 1: In this case one assumes a priori that the adjunct dynamic system is merely a *sink*: a sink is a dynamic system that *absorbs* power but *does not store* energy. Thus, in this case, $\Im_0^s(\omega) \Rightarrow 0$. From Eq. (12c), therefore, $\pi(\omega)$ is identically equal to unity:

$$\pi(\omega) = 1. \tag{13a}$$

It follows that when the adjunct dynamic system is a sink and it is attached to the master dynamic system, the external power injection remains unchanged by this attachment. (One recognizes that an adjunct dynamic system that is a sink encompasses structures that appear semi-infinite and in which reverberation cannot be established [9,27,28]. They take power, but they give none of it back!) On the other hand, the loss factor (η_s) that characterizes the adjunct dynamic system, which is in this case a sink, will yield, from Eqs. (6) and (12), the induced noise control parameter $\xi(\omega)$ to be

$$\xi(\omega) \Rightarrow \xi_0(\omega) = [\eta_0(\omega)][\eta_0(\omega) + \eta_I(\omega)]^{-1}, \quad \eta_I(\omega) = \eta_s(\omega)\Im_0^s(\omega). \tag{13b}$$

Here the induced loss factor $\eta_I(\omega)$ is the loss factor contributed to the master dynamic system by an adjunct dynamic system that is a priori a sink. In this case $\eta_I(\omega)$ is commensurate with the loss factor $\eta_0(\omega)$ that is also assumed a priori to be contributed by an attachment to a sink. From Eq. (13) one needs to recognize that if $\eta_I(\omega)$ is to be finite, $\eta_s(\omega)$ cannot be selected arbitrarily small compared with unity.

In the remaining two cases, Case 2 and Case 3, the adjunct dynamic system is not a sink. Indeed, in both cases it is assumed that the global coupling strength $\Im_0^s(\omega)$ can be rendered, by design, to exceed unity: $\Im_0^s(\omega) > 1$. (Nonetheless, as tacitly assumed, even though $\Im_0^s(\omega) > 1, [(m_s/m_0)\Im_0^s(\omega)] \ll 1$.)

Case 2: If in addition to rendering $\Im_0^s(\omega) > 1$, the loss factor $\eta_s(\omega)$ in the adjunct dynamic system is designed to highly exceed the loss factor $\eta_0(\omega)$ that is indigenous to the master dynamic system, $[\eta_s(\omega)/\eta_0(\omega)] \gg 1$, then from Eq. (12) one obtains

$$\xi(\omega) = [1 + \{\eta_s(\omega)/\eta_0(\omega)\}\Im_0^s(\omega)]^{-1}[1 + \Im_0^s(\omega)],$$
$$\xi(\omega) < [\eta_0/\eta_s(\omega)][\Im_0^s(\omega)]^{-1}[1 + \Im_0^s(\omega)] \ll 1, \tag{14a}$$

which describes a beneficial noise control. It needs to be said in this connection that were the ratio $\pi(\omega)$ of the external input power assumed to be equal to unity, as stated in Eq. (5a.1), the apparent noise control achieved under this (false) assumption, would be even more beneficial than that estimated in Eq. (14a) i.e.,

$$\xi(\omega) \Rightarrow \xi_0(\omega) \cong [1 + \{\eta_s(\omega)/\eta_0(\omega)\}\Im_0^s(\omega)]^{-1} < [\eta_0(\omega)/\eta_s(\omega)][\Im_0^s(\omega)]^{-1} \ll 1. \tag{14b}$$

Case 3: If the global coupling strength $\Im_0^s(\omega)$ can be rendered high enough, such that even if, by design, $[\eta_s(\omega)/\eta_0(\omega)] < 1$, and $[\{\eta_s(\omega)/\eta_0(\omega)\}\Im_0^s(\omega)]$ is still in excess of unity, then

$$\xi(\omega) \cong [\eta_0(\omega)/\eta_s(\omega)] > 1, \qquad [\eta_0(\omega)/\eta_s(\omega)] > \Im_0^s(\omega) > 1. \tag{15a}$$

Eq. (15a) describes a noise control *reversal*. Again, were the ratio $\pi(\omega)$ of the external input power assumed to be equal to unity, as stated in Eq. (5a.1), the noise control reversal would not emerge; namely, under this (false) assumption

$$\xi(\omega) \Rightarrow \xi_0(\omega) \cong [1 + \{\eta_s(\omega)/\eta_0(\omega)\}\Im_0^s(\omega)]^{-1} < 1, \tag{15b}$$

which is a beneficial noise control, thus, concealing the estimated noise control reversal quoted in Eq. (15a) [21,22]. (Again, it is recalled that in this consideration $[(m_s/m_0)\Im_0^s(\omega)]$ is assumed to be small compared with unity.)

A corollary to cases 2 and 3 follows. Were the adjunct dynamic system loss factor-wise similar to the master dynamic system, in the sense that $\eta_0(\omega) = \eta_s(\omega)$, the result would be that Eqs. (12a)–(12c) would assume the forms

$$\xi(\omega) \Rightarrow 1, \tag{16a}$$

$$\xi_0(\omega) = [1 + \Im_0^s(\omega)]^{-1} = [\eta_e(\omega)/\eta_v(\omega)] < 1, \tag{16b}$$

$$\pi(\omega) \Rightarrow [1 + \Im_0^s(\omega)] = [\eta_v(\omega)/\eta_e(\omega)] > 1, \tag{16c}$$

respectively. Thus, when $\eta_0(\omega) = \eta_s(\omega)$, the coupling would produce no noise control benefit; the coupling is, in this case, neutral.

It emerges in the determination of the induced noise control parameter $\xi(\omega)$, that in addition to the obvious roles played, by the indigenous loss factors $\eta_0(\omega)$ and $\eta_s(\omega)$ (and the modal masses (m_0) and (m_s)) in the master and in the adjunct dynamic systems, respectively, there is the crucial role played by either the induced loss factor $\eta_I(\omega)$ or, commensurably, by the global coupling strength $\Im_0^s(\omega)$ (cf. Eq. (16)). This is especially so when the adjunct dynamic system is not merely a sink. (One recalls that in a sink $\Im_0^s(\omega) \Rightarrow 0$ and $\eta_s(\omega)\Im_0^s(\omega) \Rightarrow \eta_I(\omega)$.) The natures and the compositions of $\eta_I(\omega)$ and $\Im_0^s(\omega)$ for a non-zero $\Im_0^s(\omega)$ are therefore, investigated next.

4. Specific evaluations of the induced loss factors and the global and modal coupling strengths

As already stated in Eq. (8), the modal coupling strength $\varsigma_0^s(\omega)$ is related to the global coupling strength $\Im_0^s(\omega)$ by the form

$$\Im_0^s(\omega) = [N_s(\omega)/N_0(\omega)]\varsigma_0^s(\omega), \quad \varsigma_0^s(\omega) = [\varepsilon_s(\omega)/\varepsilon_0(\omega)], \tag{17a}$$

$$E_0(\omega) = \Delta\omega v_0(\omega)\varepsilon_0(\omega), \quad E_s(\omega) = \Delta\omega v_s(\omega)\varepsilon_s(\omega),$$

$$N_0(\omega) = \Delta\omega v_0(\omega) = (\Delta\omega/\omega_0)[\omega_0 v_0(\omega)], \quad N_s(\omega) = \Delta\omega v_s(\omega) = (\Delta\omega/\omega_0)[\omega_0 v_s(\omega)], \tag{17b}$$

where $v_0(\omega)$ and $\varepsilon_0(\omega)$ and $v_s(\omega)$ and $\varepsilon_s(\omega)$ are the modal density and the modal stored energy in the master and in the adjunct dynamic systems, respectively, $(\Delta\omega)$ is a suitable frequency bandwidth centered about (ω) and (ω_0) is a suitable normalizing frequency [1–4]. Averaging a la Skudrzyk is implied in the likes of Eq. (17) and in some subsequent equations [20]. An interpretation that underlines Skudrzyk averaging is that the modal density $v(\omega)$ is a reasonably smooth function of frequency. This smoothness is achieved by properly extrapolating and interpolating the discrete distributions of the modal resonance frequencies. A distribution of this kind is designated (X_r); $X_r = (\omega_r/\omega_0)$, where (ω_r) is the modal resonance frequency of the (r)th mode and, again, (ω_0) is a suitable normalizing frequency. It is convenient to arrange the mode sequentially indexed, namely

$$X_r = (\omega_r/\omega_0) \leqslant (\omega_{r+1}/\omega_0) = X_{r+1}. \tag{18a}$$

In the extrapolations and interpolations, the index (r) is given a continuous connotation so that the smoothness and the sequentiality of the distribution may be defined in the form

$$X(r) = [\omega(r)/\omega_0], \quad X(r-\varepsilon) \leqslant X(r+\varepsilon)\varepsilon > 0. \tag{18b}$$

One may then define the corresponding local modal density in the form

$$[\omega_0 v(r)] = [\partial X(r)/\partial r]^{-1}. \tag{18c}$$

With this definition in place, mean-value estimates of the induced loss factor $\eta_I(\omega)$, for a typical mode in the master dynamic system, may be determined [3,4,16,23,24]. The result is

$$\eta_I(\omega) \cong (\pi/2)(\omega/\omega_0)^3[\omega_0 v_s(\omega)](m_s/m_0)C(\omega). \tag{19a}$$

The coupling factor $C(\omega)$, in Eq. (19a), is expressed in terms of the normalized coupling coefficients in the form

$$C(\omega) = [1 + \mathfrak{m}_c(\omega)]^{-1}\ C_0(\omega), \quad C_0(\omega) = [\{\mathfrak{m}_c(\omega) + \alpha_c(\omega)\}^2 + \{g(\omega)\}^2], \tag{20a}$$

$$\mathfrak{m}_c(\omega) = [m_c(\omega)/m_s], \quad \alpha_c(\omega) = [k_c(\omega)/(\omega^2 m_s)], \quad g(\omega) = [G(\omega)/\omega m_s]. \tag{20b}$$

In Figs. 2 and 3 and in Eq. (20), the vector $\{m_c(\omega), k_c(\omega), G(\omega)\}$ defines the mass, the stiffness and the gyroscopic coupling coefficients, respectively [3,4,29]. One may conveniently categorize the coupling factor $C(\omega)$ from-strong-to-weak in the form [16,18]

$$C(\omega) \cong (1/\pi), \quad \text{strong couplings}, \tag{21a}$$

$$C(\omega) \cong (\pi 10^{-2}), \quad \text{moderate couplings}, \tag{21b}$$

$$C(\omega) \cong (10^{-3}), \quad \text{weak couplings}. \tag{21c}$$

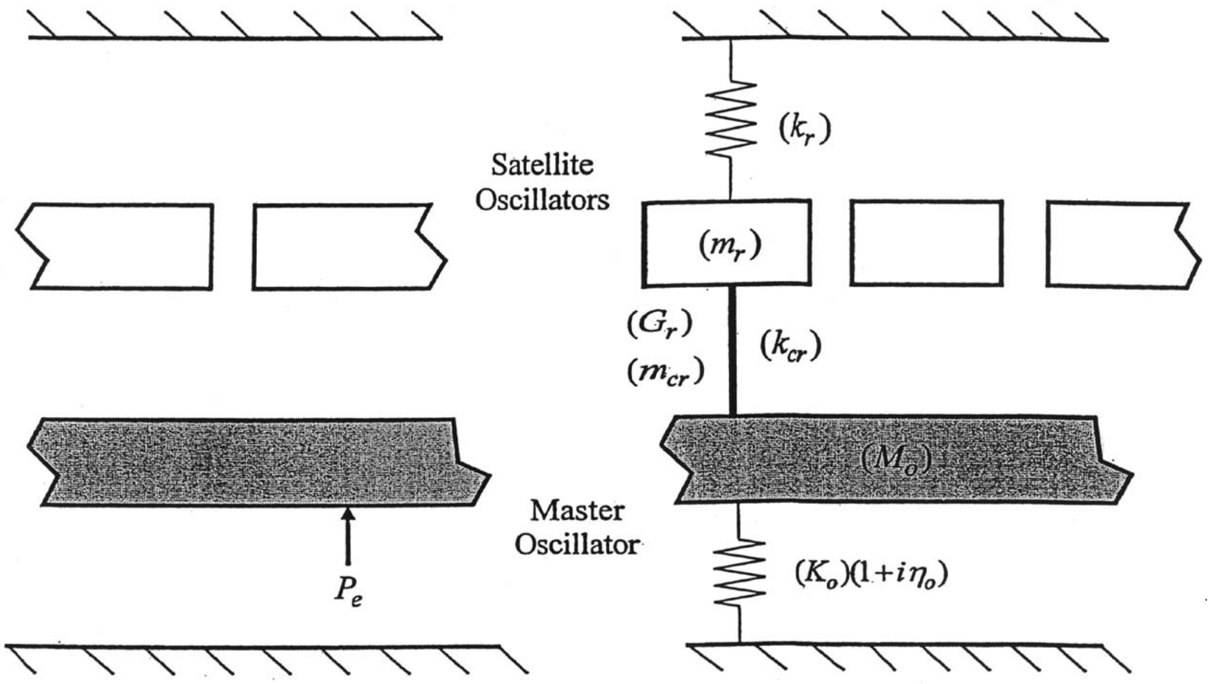

Fig. 3. A sketch of a specific complex dynamic system comprising a master dynamic system consisting of a single harmonic oscillator (a master oscillator) and an adjunct dynamic system consisting of several harmonic (satellite) oscillators. The satellite oscillators are identified by the index (r). The satellite oscillators are uncoupled to each other. Also, only the master oscillator is externally force-driven.

From Eqs. (8) and (17) one may state that

$$[N_s(\omega)m_s/N_0(\omega)m_0] = (M_s/M_0), \quad M_0 = N_0(\omega)m_0, \;\; M_s = N_s(\omega)m_s \tag{22}$$

and then, using Eq. (18a), one may recast Eq. (19a) in the form

$$\eta_I(\omega) \cong (\pi/2)(\omega/\omega_0)^3[\omega_0 v_0(\omega)](M_s/M_0)C(\omega). \tag{19b}$$

Remarkably, $\eta_I(\omega)$ appears to be independent of the loss factor $\eta_s(\omega)$ in the adjunct dynamic system. However, this independence is not free of interpretive pitfalls and has led to many erroneous statements. To straighten out these misuses, it is convenient to express $\eta_s(\omega)$ in terms of the modal overlap parameter $b_s(\omega)$ [16,23,24]. The (local) modal overlap parameter $b_s(\omega)$ is the ratio of a typical (local) modal bandwidth $[\omega\eta_s(\omega)]$ to a typical (local) separation between neighboring modes $[v_s(\omega)]^{-1}$, namely

$$b_s(\omega) = [\omega\eta_s(\omega)]v_s(\omega), \quad \eta_s(\omega) = b_s(\omega)[\omega_0 v_s(\omega)]^{-1}(\omega_0/\omega), \tag{23}$$

where the modal density $v_s(\omega)$ conforms to its definition in Eq. (18c). The convenience lies in that $b_s(\omega)$ properly determines the degree of damping that $\eta_s(\omega)$ represents, namely

$$b_s(\omega)\begin{cases} \ll 1, & \text{light damping,} \\ \cong 1, & \text{moderate damping,} \\ \gg 1, & \text{heavy damping.} \end{cases} \tag{24}$$

It transpires that the derivation of $\eta_I(\omega)$ in the form stated in Eq. (19) is predicated on a priori assigning a continuous connotation to the indices that identify the modes in the adjunct dynamic system. (Here, in the master dynamic system, a single typical mode is focused upon in the evaluations of the induced loss factor.) This assignment allows the summations over modes in the adjunct dynamic system to be replaced by integrations over the continuous indices. This process yields average (a la Skudrzyk) values for $\eta_I(\omega)$ when $b_s(\omega)$ is less than unity. (Unlike the integrations, the summations in this instance, i.e., when $b_s(\omega)$ is less than unity, yield-induced loss factor values that undulate as functions of the normalized frequency (ω/ω_0). The excursions of the undulations from the mean values are the more pronounced the lower the values of $b_s(\omega)$ are [16,23].) On the other hand, when $b_s(\omega)$ exceeds unity, the evaluation of the induced loss factors by summations over the individual modes substantially match those values derived by integrations over the continuous indices [16,23]. In this sense $b_s(\omega)$ plays a pivotal role in the interpretation of the data yielded and stated in Eq. (19) and the erroneous statements mentioned earlier are, thereby, avoided. Now, Eqs. (6b), (8), (13b), (19) and (24) may be employed to estimate the modal coupling strengths. These estimates are

$$\varsigma_0^s(\omega) \cong (\pi/2)(\omega/\omega_0)^4[\omega_0 v_s(\omega)]^2(m_s/m_0)[C(\omega)/b_s(\omega)], \quad (25a)$$

or equivalently,

$$\varsigma_0^s(\omega) \cong (\pi/2)(\omega/\omega_0)^4[\omega_0 v_0(\omega)\omega_0 v_s(\omega)](M_s/M_0)[C(\omega)/b_s(\omega)]. \quad (25b)$$

It is conducive to exemplify computations of the induced loss factor $\eta_I(\omega)$, stated in Eq. (19), and the modal coupling strength $\varsigma_0^s(\omega)$, stated in Eq. (25), by employing the complex sketched in Fig. 3 and previously investigated in Refs. [16,23].

5. Computations of the induced loss factor $\eta_I(\omega)$ and the modal coupling strength $\varsigma_0^s(\omega)$

To facilitate the computations a more definitive statement must be introduced regarding the complex dynamic system. Thus, from Fig. 3 one surmises that $N_0(\omega)$ is equal to unity and, therefore, $m_0 \equiv M_0$ and the averaging over the modes in the master dynamic system is, thereby, circumvented (cf. Eq. (17)). In addition, the resonance frequency of the master dynamic system in isolation is denoted (ω_0). The harmonic oscillators pertaining to the adjunct dynamic system are distinguished, in this case, as satellite oscillators. The number $N_s(\omega)$ of satellite oscillators may exceed unity: $N_s(\omega) \geqslant 1$ (cf. Eq. (17)). In addition, the resonance frequency of the (r)th satellite oscillator is designated (ω_r). The normalized resonance frequency distribution $X(r)$ of the satellite oscillators is defined as

$$(\omega_r/\omega_0) = [\omega(r)/\omega_0] = X(r), \quad 1 \leqslant r \leqslant N_s, \quad (26)$$

and is exhibited in Fig. 4(a). In this figure,

$$X(r) = [1 + \{1 + N_s - 2r\}(\gamma/2N_s)]^{-1/2}, \quad \gamma \cong 0.6, \quad N_s(\omega) \geqslant 1, \quad (27a)$$

and $N_s = 27$ [23,24]. Both the discrete and continuous forms of $X(r)$ are depicted in Fig. 4(a) [23]. Also, in this example it is imposed that as many satellite oscillators are with resonance frequencies that are less than the resonance frequency (ω_0), as there are those with resonance frequencies which exceed (ω_0) [16]. Similarly, the loss factor η_r that is associated with the (r)th satellite

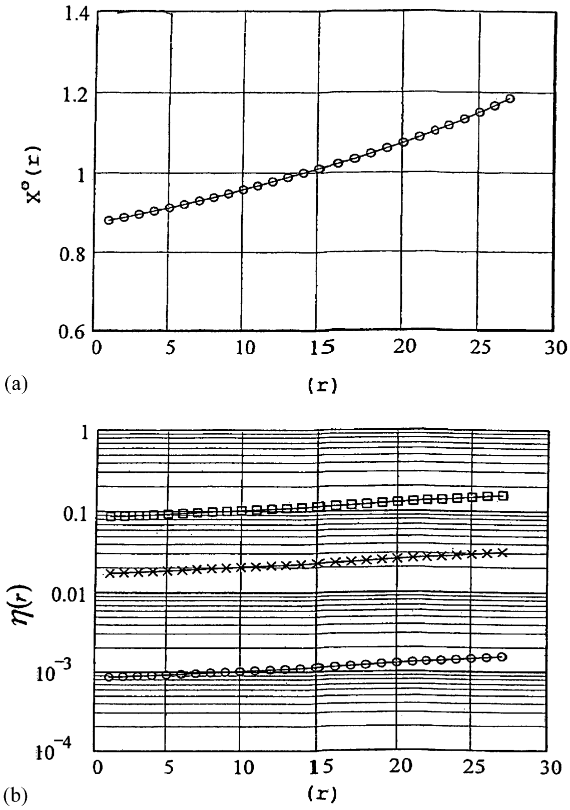

Fig. 4. (a) The normalized resonance frequency distribution $X(r)$ of the harmonic oscillators in the adjunct dynamic system (the satellite oscillators) as a function of the index (r). $X(r)$ is as stated in Eq. (29a) and $N_s = 27$. ○, Discrete; ——, continuous. (b) The localized loss factor $\eta(r)$ of the harmonic oscillator in the adjunct dynamic system (the satellite oscillators) as a function of the index (r). $\eta(r)$ as stated in Eq. (27b), $N_s = 27$ and ○, discrete; ——, continuous for $b(r) = 0.1$; ×, discrete; ——, continuous for $b(r) = 2.0$; □, discrete; ——, continuous for $b(r) = 10$.

oscillator may be cast in the form

$$\eta_r \Rightarrow \eta(r) = b(r)[X(r)]^{-1}[\partial X(r)/\partial r] = b(r)(\gamma/2N_s)[X(r)]^2, \tag{27b}$$

where $b(r)$ is a designated localized modal overlap parameter; localized at and in the vicinity of the resonance frequency (ω_r) of the (r)th satellite oscillator [16,23]. Employing Eq. (27a), with $N_s = 27$ in Eq. (27b), $\eta(r)$ is exemplified in Fig. 4(b); in this example three constant values of $b(r)$ are used namely, $b(r) = (0.1)$, (2.0) and (10) [23,24]. Again, both the discrete and the continuous forms of $\eta(r)$ are depicted in Fig. 4(b) [23].

Using the expression for the normalized resonance frequency distribution $X(r)$, as stated in Eq. (27a), one derives from Eq. (18c) the expressions

$$[\omega_0 v_s(\omega)] = N_s(\omega)(2/\gamma)(\omega_0/\omega)^3, \tag{28a}$$

$$(\Delta\omega/\omega_0)^{-1} = (2/\gamma)(\omega/\omega_0)^3 = [\omega_0 v_0(\omega)]^{-1}, \tag{28b}$$

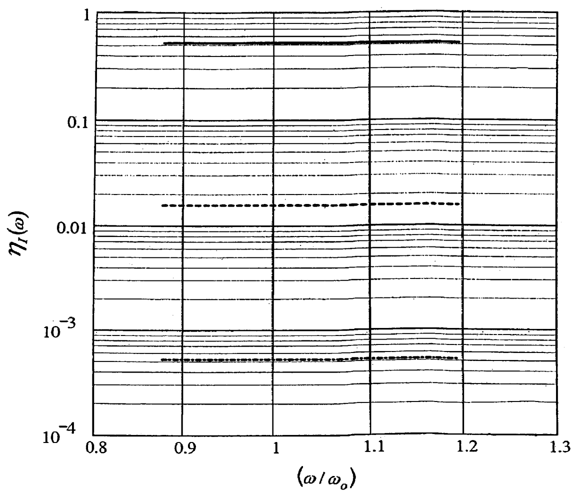

Fig. 5. The induced loss factor $\eta_I(\omega)$ as a function of the normalized frequency (ω/ω_0) for $N_s = 27$, $(M_s/M_0) = 0.1$ and for three values of the coupling factor $C(\omega)$; $C(\omega) = 1.0$ (solid curve), $C(\omega) = 3 \times 10^{-2}$ (dash curve), and $C(\omega) = 10^{-3}$ (dash–dot curve). The frequency bandwidth $[\Delta(\omega)/\omega_0] \cong (\gamma/2) = 0.3$.

where again $N_0(\omega) = 1$. From Eqs. (19) and (28) one obtains

$$\eta_I(\omega) = (\pi/2)(2/\gamma)(M_s/M_0)C(\omega). \tag{29}$$

The values for the induced loss factor $\eta_I(\omega)$, as a function of the normalized frequency (ω/ω_0) in the appropriate range of frequency, are depicted in Fig. 5. The normalized range of frequency is given by

$$[\Delta(\omega)/\omega_0] \cong [1 - (\gamma/2)]^{-1/2} - [1 + (\gamma/2)]^{-1/2} \cong (\gamma/2). \tag{30}$$

Again, in Fig. 5, $N_s = 27$ and the global mass ratio (M_s/M_0) is set equal to one tenth (1/10). There are three curves in Fig. 5, the solid curve corresponds to a strong coupling with $C(\omega) = 1$, the dash curve corresponds to a moderate coupling with $C(\omega) = 3 \times 10^{-2}$ and the dash–dot curve corresponds to a weak coupling with $C(\omega) = 10^{-3}$ (cf. Eq. (21)). The normalized range of frequency in Fig. 5 is set by $\gamma = 0.6$. From Eq. (29) it is clear that the induced loss factor increases with an increase in the global mass ratio (M_s/M_0), with an increase in the coupling factor $C(\omega)$ and with a decrease in the normalized range of frequency, namely, a decrease in (γ); notwithstanding that (γ) must be chosen less than unity. It may be of interest to contrast the induced loss factor $\eta_I(\omega)$ with the corresponding assigned values for the indigenous loss factor $\eta_s(\omega)$ in the adjunct dynamic system. The expression for $\eta_s(\omega)$ is derived from Eqs. (23) and (28) to be

$$\eta_s(\omega) = b_s(\omega)(\omega/\omega_0)^2[\gamma/2N_s(\omega)]. \tag{31}$$

The loss factor $\eta_s(\omega)$ is depicted as a function of (ω/ω_0), for $\gamma = 0.6$ and $N_s(\omega) = 27$, in Fig. 6. The three curves in Fig. 6 pertain to $b_s(\omega) = 0.1$ (solid), $b_s(\omega) = 2.0$ (dash) and $b_s = 10$ (dash–dot). The results in Figs. 5 and 6 may be compared as a bench mark. It is clear, for example, that $\eta_I(\omega)$ definitively exceeds $\eta_s(\omega)$ and when coupling is strong $\Im_0^s(\omega)$ exceeds unity (cf. Eq. (6)).

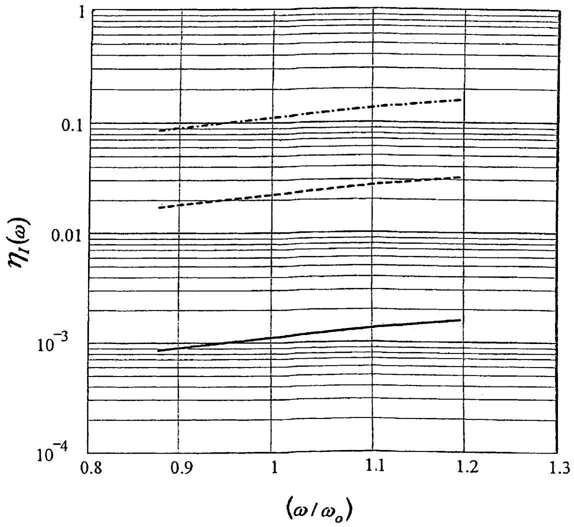

Fig. 6. The indigenous loss factor $\eta_s(\omega)$ in the adjunct dynamic system as a function of the normalized frequency (ω/ω_0) for $N_s = 27$, $(M_s/M_0) = 0.1$ and for three values of the modal overlap parameter $b_s(\omega)$; $b_s(\omega) = 0.1$ (solid), $b_s(\omega) = 2.0$ (dash) and $b_s(\omega) = 10.0$ (dash–dot). The frequency bandwidth $[\Delta(\omega)/\omega_0] \cong (\gamma/2) = 0.3$.

Using Eqs. (28) and (31) in Eq. (25) one obtains for the modal coupling strength $\varsigma_0^s(\omega)$ the expressions

$$\varsigma_0^s(\omega) \cong [\pi/2b_s(\omega)](\omega_0/\omega)^2(2/\gamma)^2[N_s(\omega)](m_s/m_0)[C(\omega)], \tag{32a}$$

$$\varsigma_0^s(\omega) \cong (\pi/2)(\omega_0/\omega)^2(2/\gamma)^2(M_s/M_0)[C(\omega)/b_s(\omega)], \quad N_0(\omega) = 1. \tag{32b}$$

The values for the modal coupling strength, as a function of the normalized frequency (ω/ω_0), in the appropriate range of frequency, as defined in Eq. (30), are depicted in Fig. 7. Again, in Fig. 7 the global mass ratio is set equal to one-tenth $(M_s/M_0) = 0.1$, and the normalized range of frequency is set by $\gamma = 0.6$. Clearly, as is the induced loss factor $\eta_I(\omega)$, the modal coupling strength $\varsigma_0^s(\omega)$ increases with increase in the global mass ratio (M_s/M_0) and the coupling factor $C(\omega)$. Also, as does $\eta_I(\omega)$, $\varsigma_0^s(\omega)$ increases as (γ) decreases. Significantly, however, the modal coupling strength $\varsigma_0^s(\omega)$ is inversely proportional to the modal overlap parameter $b_s(\omega)$. The modal coupling strength $\varsigma_0^s(\omega)$, as a function of normalized frequency (ω/ω_0), is depicted in Fig. 7. The ordinate in Fig. 7 is designed to accommodate three sets of curves. Each set pertains to a particular coupling factor $C(\omega)$; in the first set $C(\omega) = 1$, in the second $C(\omega) = 3 \times 10^{-2}$ and in the third $C(\omega) = 10^{-3}$. The ordinate for each set spans four decades. For the first set the span is from 10^{-2} to 10^2. For the second set the span is from 3×10^{-4} to 3 and for the third from 10^{-5} to 10^{-1}. With this understanding all three sets are depicted in Fig. 7. In this figure only the span of the first set is stated explicitly; the spans for the other two sets are merely implicit. In each set of Fig. 7 three curves are depicted; the solid curve pertains to a $b_s(\omega)$ that is equal to one-tenth (0.1), the dash curve pertains to a $b_s(\omega)$ that is equal to two and the dash–dot curve pertains to a $b_s(\omega)$ that is equal to 10. In the light of the statistical energy analysis (SEA), a major observation emerges from Fig. 7; the observation is discussed briefly in the next section [3,4].

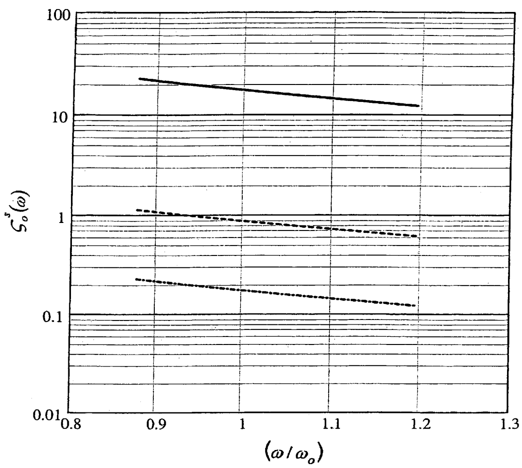

Fig. 7. The modal coupling strengths $\varsigma_0^s(\omega)$, as a function of the normalized frequency (ω/ω_0), for three values of the modal overlap parameter $b_s(\omega); b_s\ (\omega) = 0.1$ (solid curve), $b_s(\omega) = 2.0$ (dash curve) and $b_s(\omega) = 10.0$ (dash–dot curve). In these curves $N_s = 27$ and $(M_s/M_0) = 0.1$. The explicit ordinate span of four (4) decades from 10^{-2} to 10^2 is for a coupling factor of $C(\omega) = 1$. For a coupling factor of $C(\omega) = 3 \times 10^{-2}$ and of $C(\omega) = 10^{-3}$ an ordinate span of four (4) decades is implicitly understood to extend instead from 3×10^{-4} to 3 and from 10^{-5} to 10^{-1}, respectively. In these implicit renderings everything else in the graphs remain intact.

6. Validity of SEA

An observation emerges when the modal coupling strength $\varsigma_0^{sea}(\omega)$ is estimated via (SEA) rather than estimated in the manner described herein e.g., as stated in Eq. (32) [16]. In terms of SEA the net power $\Pi_s(\omega)$ that invades the adjunct dynamic system is given by

$$\Pi_s(\omega) = \eta_{s0}(\omega)[\omega E_0(\omega)] - \eta_{0s}(\omega)[\omega E_s(\omega)],$$
$$[\eta_{s0}(\omega)/\eta_{0s}(\omega)] = [N_s(\omega)/N_0(\omega)] = [v_s(\omega)/v_0(\omega)], \tag{33}$$

where $\eta_{s0}(\omega)$ and $\eta_{0s}(\omega)$ are the coupling loss factors from the master dynamic system to the adjunct dynamic system and vice versa, respectively [3,4]. From Eq. (33) one obtains

$$[\Pi_s\,(\omega)/\omega E_0(\omega)]^{sea} = \eta_{s0}(\omega)[1 - \varsigma_0^{sea}(\omega)],$$
$$\varsigma_0^{sea}(\omega) = \eta_{0s}(\omega)[\eta_s(\omega) + \eta_{0s}(\omega)]^{-1}, \tag{34a}$$

or equivalently

$$\left[\Pi_s\,(\omega)/\omega E_0(\omega)\right]^{sea} = \eta_I^{sea}(\omega) = \eta_s(\omega)\eta_{s0}(\omega)[\eta_s(\omega) + \eta_{0s}(\omega)]^{-1} = \eta_s(\omega)\Im_0^{sea}(\omega),$$
$$\Im_0^{sea}(\omega) = [N_s(\omega)/N_0(\omega)]\varsigma_0^{sea}(\omega), \tag{34b}$$

where the superscript (*sea*) indicates that the estimates so superscripted and those enclosed in the square brackets are a la SEA [3,4]. It is clearly evident from Eq. (34a) that the modal coupling strength $\varsigma_0^{sea}(\omega)$ must, by definition, lie below unity, namely

$$\varsigma_0^{sea}(\omega) < 1. \tag{35}$$

Eq. (35) is a tenet of the statistical energy analysis (SEA) [3,4,25].

It is observed from Eq. (32) that no such restriction is imposed on $\varsigma_0^s(\omega)$. Indeed, for the complex dynamic system depicted in Fig. 3, Fig. 7 exhibits clearly that $\varsigma_0^s(\omega)$ may exceed unity. For $\varsigma_0^s(\omega)$, which is determined via the energy analysis (EA), to be compatible with $\varsigma_0^{sea}(\omega)$, which is determined via the statistical energy analysis (SEA), necessarily requires that $\varsigma_0^s(\omega)$ remains below unity. From Eq. (34) this requirement may be expressed in terms of a minimal value $[b_s(\omega)]_M$ for the modal overlap parameter $b_s(\omega)$, namely

$$b_s(\omega) > [b_s(\omega)]_M = (\pi/2)(\omega_0/\omega)^2(4/\gamma^2)(M_s/M_0)C(\omega). \quad (36)$$

The parameter $[b_s(\omega)]_M$ necessarily designates the minimum value of the modal overlap parameter that is needed to validate the statistical energy analysis [14,16]. Again, for the complex dynamic system depicted in Fig. 3 and corresponding to Fig. 7, the values of $[b_s(\omega)]_M$, as a function of (ω/ω_0), are exhibited in Fig. 8 for three values of the coupling factor $C(\omega)$: $C(\omega) = 1$ (dash–dot), $C(\omega) = 3 \times 10^{-2}$ (dash) and $C(\omega) = 10^{-3}$ (solid). It is observed, in Fig. 8, that (SEA) is validated for values of $b_s(\omega)$ that are less than unity only for coupling factors that are small compared with unity. It is, thus, speculated that the use of (SEA) may be strained when dealing with strong and even with moderate couplings. The validation of (SEA) may be called to question when the loss factors that are associated with the adjunct dynamic systems assume arbitrarily low values and the coupling factors assume high values [16–19]. (This idiosyncrasy between (EA) and (SEA) reminds one of the noise control reversal discussed in Section 3 and is, of course, related to it [26].) In this connection one is reminded that undulations in the non-averaged values in the response quantities, of a complex dynamic system are suppressed only when $b_s(\omega)$ exceeds unity [16,23]. However, the absence or the presence of undulations in that non-average response do not bear on the criterion for the validity of (SEA); (SEA) is validated as long as the modal overlap parameter

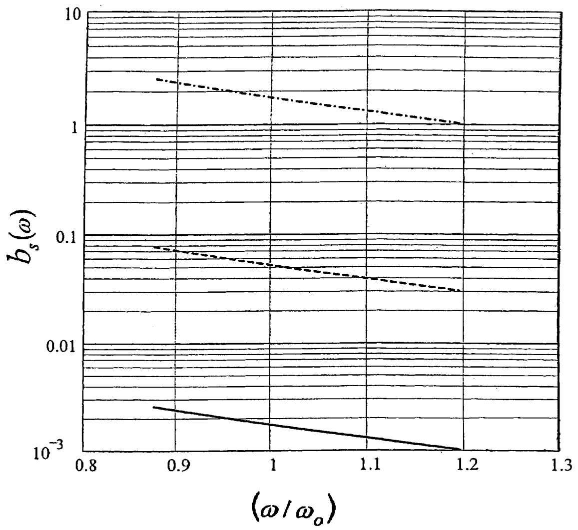

Fig. 8. The minimum value of the modal overlap parameter $[b_s(\omega)]_M$, as a function of the normalized frequency (ω/ω_0), for $N_s = 27$ and $(M_s/M_0) = 0.1$ and for three values of the coupling factor $C(\omega)$; $C(\omega) = 1$ (dash–dot), $C(\omega) = 3 \times 10^{-2}$ (dash) and $C(\omega) = 10^{-3}$ (solid). The statistical energy analysis is *necessarily* valid for corresponding values of the modal overlap parameter $b_s(\omega)$ that exceed the minimum value shown in this figure.

$b_s(\omega)$ exceeds the corresponding minimum value of this parameter, i.e., as long as $b_s(\omega) > [b_s(\omega)]_M$ is valid. Indeed, the employment of (SEA) may be valid for values of both $[b_s(\omega)]_M$ and $b_s(\omega)$ that may be less than unity. Eq. (36) makes clear that the validity of (SEA) merely requires that $b_s(\omega)$ exceeds $[b_s(\omega)]_M$. The validity does not demand that undulations be absent in the non-averaged values of the response quantities. Of course (SEA) yields largely average response quantities, which are commensurably averaged a la Skudrzyk [3,4,20].

Acknowledgements

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